

# 6-1

## The Polygon Angle-Sum Theorems

**Common Core State Standards**

**G-SRT.B.5** Use congruence . . . criteria to solve problems and prove relationships in geometric figures.

**MP 1, MP 3**

**Objectives** To find the sum of the measures of the interior angles of a polygon  
To find the sum of the measures of the exterior angles of a polygon



If you can find a pattern, you won't have to draw all those diagonals.



### Getting Ready!

Sketch a convex pentagon, hexagon, and heptagon. For each figure, draw all the diagonals you can from one vertex. What conjecture can you make about the relationship between the number of sides of a polygon and the number of triangles formed by the diagonals from one vertex?



**MATHEMATICAL PRACTICES**

The Solve It is related to a formula for the sum of the interior angle measures of a polygon. (In this textbook, a polygon is convex unless otherwise stated.)

**Essential Understanding** The sum of the interior angle measures of a polygon depends on the number of sides the polygon has.

By dividing a polygon with  $n$  sides into  $(n - 2)$  triangles, you can show that the sum of the interior angle measures of any polygon is a multiple of 180.



### Lesson Vocabulary

- equilateral polygon
- equiangular polygon
- regular polygon



### Theorem 6-1 Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an  $n$ -gon is  $(n - 2)180$ .

### Think

How many sides does a heptagon have?  
A heptagon has 7 sides.



### Problem 1 Finding a Polygon Angle Sum

What is the sum of the interior angle measures of a heptagon?

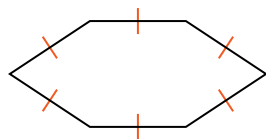
$$\begin{aligned} \text{Sum} &= (n - 2)180 && \text{Polygon Angle-Sum Theorem} \\ &= (7 - 2)180 && \text{Substitute 7 for } n. \\ &= 5 \cdot 180 && \text{Simplify.} \\ &= 900 \end{aligned}$$

The sum of the interior angle measures of a heptagon is 900.

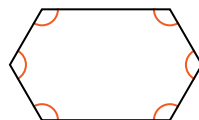


- Got It?** 1. a. What is the sum of the interior angle measures of a 17-gon?  
b. **Reasoning** The sum of the interior angle measures of a polygon is 1980. How can you find the number of sides in the polygon?

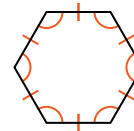
An **equilateral polygon** is a polygon with all sides congruent.



An **equiangular polygon** is a polygon with all angles congruent.



A **regular polygon** is a polygon that is both equilateral and equiangular.



take note

### Corollary to the Polygon Angle-Sum Theorem

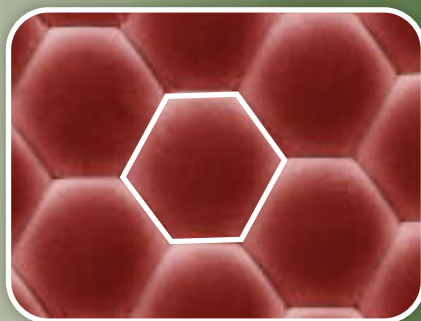
The measure of each interior angle of a regular  $n$ -gon is  $\frac{(n-2)180}{n}$ .

You will prove the Corollary to the Polygon Angle-Sum Theorem in Exercise 43.



### Problem 2 Using the Polygon Angle-Sum Theorem STEM

**Biology** The common housefly, *Musca domestica*, has eyes that consist of approximately 4000 facets. Each facet is a regular hexagon. What is the measure of each interior angle in one hexagonal facet?



$$\begin{aligned}\text{Measure of an angle} &= \frac{(n-2)180}{n} \\ &= \frac{(6-2)180}{6} \\ &= \frac{4 \cdot 180}{6} \\ &= 120\end{aligned}$$

Corollary to the Polygon Angle-Sum Theorem

Substitute 6 for  $n$ .

Simplify.

The measure of each interior angle in one hexagonal facet is 120.



**Got It?** 2. What is the measure of each interior angle in a regular nonagon?

### Think

How does the word **regular** help you answer the question?

The word *regular* tells you that each angle has the same measure.

## Think

**How does the diagram help you?**  
You know the number of sides and four of the five angle measures.



### Problem 3 Using the Polygon Angle-Sum Theorem

What is  $m\angle Y$  in pentagon *TODAY*?

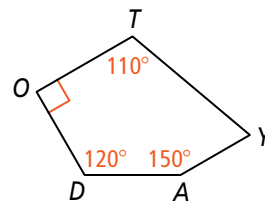
Use the Polygon Angle-Sum Theorem for  $n = 5$ .

$$m\angle T + m\angle O + m\angle D + m\angle A + m\angle Y = (5 - 2)180$$

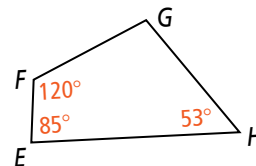
$$110 + 90 + 120 + 150 + m\angle Y = 3 \cdot 180 \quad \text{Substitute.}$$

$$470 + m\angle Y = 540 \quad \text{Simplify.}$$

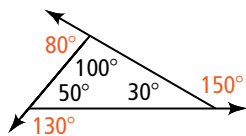
$$m\angle Y = 70 \quad \text{Subtract 470 from each side.}$$



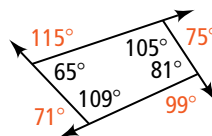
**Got It?** 3. What is  $m\angle G$  in quadrilateral *EFGH*?



You can draw exterior angles at any vertex of a polygon. The figures below show that the sum of the measures of the exterior angles, one at each vertex, is 360.



$$80 + 150 + 130 = 360$$



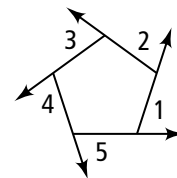
$$115 + 75 + 99 + 71 = 360$$

Take note

### Theorem 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon,  $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$ .



You will prove Theorem 6-2 in Exercise 39.

## Think

**What kind of angle is  $\angle 1$ ?**  
Looking at the diagram, you know that  $\angle 1$  is an exterior angle.

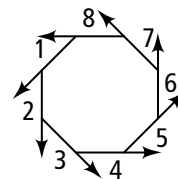


### Problem 4 Finding an Exterior Angle Measure

What is  $m\angle 1$  in the regular octagon at the right?

By the Polygon Exterior Angle-Sum Theorem, the sum of the exterior angle measures is 360. Since the octagon is regular, the interior angles are congruent. So their supplements, the exterior angles, are also congruent.

$$\begin{aligned} m\angle 1 &= \frac{360}{8} && \text{Divide 360 by 8, the number of sides in an octagon.} \\ &= 45 && \text{Simplify.} \end{aligned}$$



**Got It?** 4. What is the measure of an exterior angle of a regular nonagon?



## Lesson Check

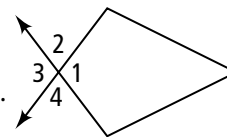
### Do you know HOW?

1. What is the sum of the interior angle measures of an 11-gon?
2. What is the sum of the measures of the exterior angles of a 15-gon?
3. Find the measures of an interior angle and an exterior angle of a regular decagon.

### Do you UNDERSTAND?



4. **Vocabulary** Can you draw an equiangular polygon that is not equilateral? Explain.
5. **Reasoning** Which angles are the exterior angles for  $\angle 1$ ? What do you know about their measures? Explain.
6. **Error Analysis** Your friend says that she measured an interior angle of a regular polygon as 130. Explain why this result is impossible.

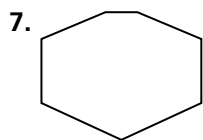


## Practice and Problem-Solving Exercises



### A Practice

Find the sum of the interior angle measures of each polygon.



7.

8. 35-gon

9. 14-gon

10. 20-gon

11. 1002-gon

← See Problem 1.

Find the measure of one interior angle in each regular polygon.



12.



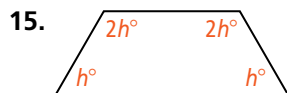
13.



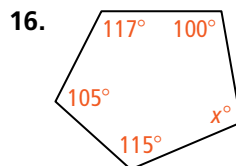
14.

← See Problem 2.

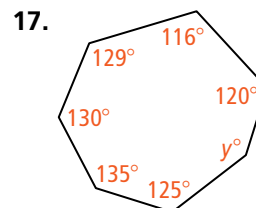
**Algebra** Find the missing angle measures.



15.



16.



17.

← See Problem 3.

Find the measure of an exterior angle of each regular polygon.

18. pentagon

19. 36-gon

20. 18-gon

21. 100-gon

← See Problem 4.

### B Apply

The sum of the interior angle measures of a polygon with  $n$  sides is given. Find  $n$ .

22. 180

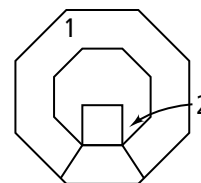
23. 1080

24. 1980

25. 2880

- © 26. **Open-Ended** Sketch an equilateral polygon that is not equiangular.

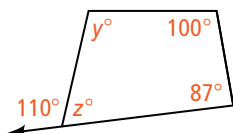
27. **Stage Design** A theater-in-the-round allows for a play to have an audience on all sides. The diagram at the right shows a platform constructed for a theater-in-the-round stage. What type of regular polygon is the largest platform? Find the measure of each numbered angle.



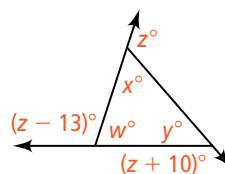
- © 28. **Think About a Plan** A triangle has two congruent interior angles and an exterior angle that measures 100. Find two possible sets of interior angle measures for the triangle.
- How can a diagram help you?
  - What is the sum of the angle measures in a triangle?

**Algebra** Find the value of each variable.

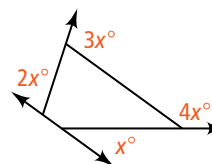
29.



30.



31.



The measure of an exterior angle of a regular polygon is given. Find the measure of an interior angle. Then find the number of sides.

32. 72

33. 36

34. 18

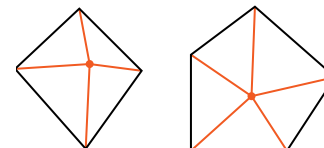
35. 30

36.  $x$

**Packaging** The gift package at the right contains fruit and cheese. The fruit is in a container that has the shape of a regular octagon. The fruit container fits in a square box. A triangular cheese wedge fills each corner of the box.



37. Find the measure of each interior angle of a cheese wedge.
- © 38. **Reasoning** Show how to rearrange the four pieces of cheese to make a regular polygon. What is the measure of each interior angle of the polygon?
39. **Algebra** A polygon has  $n$  sides. An interior angle of the polygon and an adjacent exterior angle form a straight angle.
- What is the sum of the measures of the  $n$  straight angles?
  - What is the sum of the measures of the  $n$  interior angles?
  - Using your answers above, what is the sum of the measures of the  $n$  exterior angles?
  - What theorem do the steps above prove?
- © 40. **Reasoning** Your friend says she has another way to find the sum of the interior angle measures of a polygon. She picks a point inside the polygon, draws a segment to each vertex, and counts the number of triangles. She multiplies the total by 180, and then subtracts 360 from the product. Does her method work? Explain.
41. **Algebra** The measure of an interior angle of a regular polygon is three times the measure of an exterior angle of the same polygon. What is the name of the polygon?



## Challenge

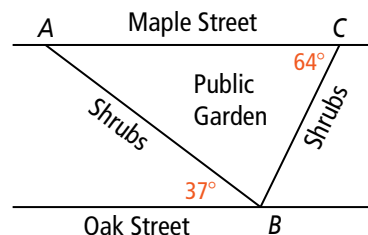
42. **Probability** Find the probability that the measure of an interior angle of a regular  $n$ -gon is a positive integer when  $n$  is an integer and  $3 \leq n \leq 12$ .
43. a. In the Corollary to the Polygon Angle-Sum Theorem, explain why the measure of an interior angle of a regular  $n$ -gon is given by the formulas  $\frac{180(n-2)}{n}$  and  $180 - \frac{360}{n}$ .
- b. Use the second formula to explain what happens to the measures of the interior angles of regular  $n$ -gons as  $n$  becomes a large number. Explain also what happens to the polygons.
44.  $ABCDEFGHJK$  is a regular decagon. A ray bisects  $\angle C$ , and another ray bisects  $\angle D$ . The two rays intersect in the decagon's interior. Find the measure of the acute angles formed by the intersecting rays.

## Standardized Test Prep

### GRIDDED RESPONSE

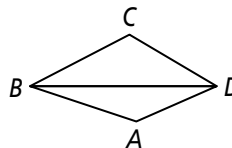
## SAT/ACT

45. The car at each vertex of a Ferris wheel holds a maximum of five people. The sum of the interior angle measures of the Ferris wheel is 7740. What is the maximum number of people the Ferris wheel can hold?
46. A rectangle and a square have equal areas. The rectangle has length 9 cm and width 4 cm. What is the perimeter of the square, in centimeters?
47. The Public Garden is located between two parallel streets: Maple Street and Oak Street. The garden faces Maple Street and is bordered by rows of shrubs that intersect Oak Street at point  $B$ . What is  $m\angle ABC$ , the angle formed by the shrubs?
48.  $\triangle ABC \cong \triangle DEF$ . If  $m\angle A = 3x + 4$ ,  $m\angle C = 2x$ , and  $m\angle E = 4x + 5$ , what is  $m\angle B$ ?



## Mixed Review

49. If  $\overline{AB} \cong \overline{CB}$  and  $m\angle ABD < m\angle CBD$ , which is longer,  $\overline{AD}$  or  $\overline{CD}$ ? Explain.



See Lesson 5-7.

Name the property that justifies each statement.

50.  $4(2a - 3) = 8a - 12$

51.  $\overline{RS} \cong \overline{RS}$

See Lesson 2-5.

52. If  $\angle 1 \cong \angle 4$ , then  $\angle 4 \cong \angle 1$ .

**Get Ready!** To prepare for Lesson 6-2, do Exercises 53 and 54.

Use the figure below.

See Lessons 4-1 and 4-3.

53. Name the postulate or theorem that justifies  $\triangle EFG \cong \triangle GHE$ .

54. Complete each statement.

a.  $\angle FEG \cong \blacksquare$

b.  $\angle EFG \cong \blacksquare$

c.  $\angle FGE \cong \blacksquare$

d.  $\overline{EF} \cong \blacksquare$

e.  $\overline{FG} \cong \blacksquare$

f.  $\overline{GE} \cong \blacksquare$

